ESTIMATE OF ERROR UNDER DIFFERENT DISTRIBUTIONS WITH THE GENERALIZED AUTOREGRESSIVE CONDITIONAL HETEROSCEDASTICITY

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Abstract: This study examines error of estimate under different distributions with the generalized autoregressive conditional heteroscedasticity. The naira exchange rate in Nigeria has exhibited the features of continuous depreciation and instability, this instability and continuous depreciation of the naira in the foreign exchange market has resulted in declines in the standard of living of the populace. Empirical results further reveal that the GARCH model achieved under three different error distributions for all the currencies, are then compared using the minimum value of the Akaike information criterion (AIC), Hannan-Quinn criterion (HIC), and Schwarz information criterion (SIC). The result shows that PARCH (1, 1) model with asymmetric order 1 under the Generalized error distribution (GED) proved to be the best model which reduces the persistence of volatility clustering in the Nigeria exchange market.

Keywords: Exchange rate volatility, heteroscedasticity, GARCH model, stationarity

Introduction

The concept of exchange rate has always assumed a central position in economic development literature and growth strategies on structural adjustment and macroeconomic stabilization. Presently, exchange rate is almost of flexible variable even in most developing countries (Frimpong and Oteng-Abayie, 2006). Exchange rate system has evolved over time. In 1959, the system of full convertibility of all major currencies was set up till 1968 (Gozgor and Nokey, 2011). The financial market tension of 1968 forced the separation of private and official markets for gold, ending the private convertibility of the dollar into gold at a fixed price. The system continued as a dollar standard with fixed parties until 1971 (Tripathy and Gil-Alana, 2010). However, the United State suspended the gold convertibility of the dollar for official borrowers in August 1971.

Most major currencies began to float against the dollar. Furthermore, by December 1971, the US dollar was devalued by around 9.1 percent and intervention exchange rate management and capital inflows. Limits were widened from 1 ± percent to 25.2 ± percent. Within the exception of the pound sterling, these parities held until 1973. Boughton (1993) admits that the system collapsed in 1972. Bollerslev (1990) and Vee *et al.* (2011), however, introduces a generalized ARCH (GARCH) process that allows for a more manageable lag structure. Bollerslev (1990) proposed a multivariate time series model with time varying conditional variances and co–variances but with conditional correlation. The validity of the model was illustrated for a set of five European/US dollar exchange rates (Wang and Barrett, 2002).

Engle (1982) noted that although OLS maintains its optimality properties, the maximum likelihood is more efficient in estimating the parameters of ARCH models. The ARCH/GARCH literature had recently focused on analysing volatility of high–frequency data and their benefits (Engle, 2002; Andersen, 2000). However, Teräsvirta (2009) reviews several univariate models of conditional heteroscedasticity and reports that GARCH models tend to exaggerate volatility persistence. Pacelli (2012) finds that, when using GARCH (1,1), GARCH models can better forecast exchange rate dynamics. Many studies support the idea of using the GARCH approach to proxy volatility, perhaps due to the existence of a non-constant variance.

Materials and Methods

The Arch model assumptions

In the ARCH model,

$$\varepsilon_t = h_t^{\frac{1}{2}} v_t, \quad V_t \approx iid(0,\sigma^2) \qquad \ldots\ldots\ldots\ldots(2.1)$$

Suppose that the $V'S$ are normally distributed. Then

- $\varepsilon_t|\varepsilon_{t-1},\ldots$ is normally distributed
- ε_t is white noise but not normally distributed – thicker tails
- we can use the normal likelihood function to estimate the model

Recall that part of the interest in ARCH models comes from the observation that conditional heteroskedasticity seems to be a common feature of (high frequency) financial time series data.

ARCH (p) Model Specification:

$$\sigma_t^2 = \omega + (\sum_{i=1}^{q} \alpha_i L^i)\varepsilon_t^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.2)$$

The ARCH model can describe volatility clustering because the conditional variance of ε_t is an increasing function of $\varepsilon^2{}_{t-1}$. Consequently, if ε_{t-1} was large in absolute value, σ_t^2 and thus ε_t is expected to be large (in absolute value) as well. The unconditional variance of ε_t exists if $\omega \succ 0$ and $\sum_{i=1}^{q} \alpha_i < 1$ is given by

$$\sigma^2 \equiv E\left[E(\varepsilon_t^2|\Omega_{t-1})\right] = \frac{\omega}{1-\sum_{i=1}^{q}\alpha_i} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(2.3)$$

The Garch model

Assumption of Garch

Let us define

$$\sigma^2_{\varepsilon t}(\theta) = \omega + \alpha \sum_{k=0}^{\infty} \beta^k \varepsilon^2_{0t-1-k} \quad \text{……………(2.4)}$$

The process $\sigma^2_{\mu t}$ is a strictly stationary model of the conditional variance which assumes an infinite history of the observed data. The process $\sigma^2_{\varepsilon t}$ is in fact identical to the process $\sigma^2_{\mu t}$ except that it is expressed as a function of the true innovations ε_{0t} instead of the residuals ε_t.

We suppose that the following conditions on the process Z_t hold:

(1) Z_t is a sequence of independent and identical distributed random variables such that $EZ_t = 0$;

(2) Z^2_t is nondegenerate;

(3) for some $\delta > 0$ exists $S_\delta < \infty$ such that $E\left[Z_t^{2+\delta}\right] \leq S_\delta < \infty$;

(4) $E\left[in\left(\beta_0 + \alpha_0 Z_t^2\right)\right] < 0$;

(5) θ_0 is in the interior of Θ

(6) if for some t holds

$$\sigma^2_{0t}(\theta) = c_0 + \sum_{k=1}^{\infty} C_k \varepsilon^2_{t-k}$$

$$\sigma^2_{0t}(\theta) = c_0^* + \sum_{k=1}^{\infty} C_k^* \varepsilon^2_{t-k}$$

then $c_1^* = c_i^*$ for every $1 \leq i < \infty$.

Autoregressive conditional heteroscedastic (ARCH) model of order q can be written as follows:

$$\varepsilon_t = z_t \sigma_t$$

$$\sigma^2_t = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon^2_{t-1,}$$

$$\sigma^2 = \omega + \alpha_1 \varepsilon^2_{t-1} + \ldots + \alpha_q \varepsilon^2_{t-q} = \omega + \sum_{i=1}^{q} (\alpha_1 \varepsilon^2_{t-1}) \quad \text{… (2.5)}$$

where Z_t = is an independently and identically distributed process with $E(Z_t) = 0$ and $Var(Z_t) = 1$. The model assumes that ε_t is serially uncorrelated, and mean zero, with time varying conditional variance, σ^2_t.

The Generalized ARCH (GARCH) is an extension of the ARCH model. When modelling using ARCH, there might be a need for a large value of the lag *p*, hence a large number of parameters. A GARCH model may contain fewer parameters as compared to an ARCH model, and thus a GARCH model may be preferred to an ARCH model.

The GARCH (p, q) model can be expressed as:

$$\sigma^2_t = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon^2_{t-1} + \sum_{j=1}^{p} \beta_j \sigma^2_{t-j.} \quad \text{……..(2.6)}$$

Where: ω = the constant term; α_i =ARCH terms i.e. volatility shocks from prior periods; β_j =GARCH terms i.e. the persistence of volatility; p =the number of lagged conditional variance terms (σ^2); q =the number of lagged errors (ε^2).

To ensure that σ^2_t is strictly positive we have to impose some restrictions with respect to the parameters in the conditional variance equation:

$$\omega \succ 0, \alpha_i \geq 0, \; for \quad i = 1, q, \beta_j \geq 0, \; for \quad j = 1, p$$

$$and \sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j \prec 1 \quad \text{……….(2.7)}$$

Using the lag (or backshift) operator *L*, the GARCH (p, q) model becomes:

$\sigma^2_t = \omega + \alpha(L)\varepsilon^2_t + \beta(L)\sigma^2_t$, (i.e. good news and bad news have a declining impact on future volatility).

Data analysis

The best Garch model achived under different error distribution

Garch model achived under normal Gaussian distribution

All the models are then compared using the smallest values of the Akaike information criterion (AIC) Hannan-Quinn criterion (HIC) and Schwarz information criterion (SIC) during estimation to select the best model which is better for exchange rate volatility if more than one model with given specifications is found to be significant.

Method: ML ARCH - Normal Gaussian distribution (BFGS / Marquardt steps)

	GARCH	TGARCH	EGARCH	PARCH	CGARCH	GARCH	TGARCH	EGARCH	PARCH	CGARCH
Model	(1, 1)	(1, 1)	(1, 1)	(1, 1)	(1, 1)	(2, 2)	(2, 2)	(2, 2)	(2, 2)	(2, 2)
Akaike info criterion	6.665184	6.664584	6.672596	6.661307	6.639231	7.026417	7.074678	6.899644	6.944327	6.835440
Schwarz criterion	6.689139	6.691961	6.696551	6.688685	6.670030	7.057216	7.108899	6.933866	6.985393	6.869661
Hannan-Quinn criterion	6.674089	6.674761	6.681501	6.671485	6.650680	7.037866	7.087400	6.912366	6.959593	6.848162
Adjusted R-squared	0.973014	0.973049	0.972663	0.972935	0.972979	0.972550	0.972869	0.974363	0.972543	0.975130

Result discussion of Garch models under normal Gaussian distribution

From the above result comparison between the GARCH models under the normal Gaussian distribution, the selection criterion were based on Selecting the Minimum AIC, SIC and HIC, from the based result above Akaike Information Criterion, Schwarz Criterion and Hannan-Quinn Criterion that the best models have proved to be CGARCH (1, 1). With the minimum estimated AIC of 6.639231, SIC estimated to be 6.670030, and HIC estimated to be 6.650680. Therefore, selecting the best model using the normal Gaussian error distribution is the CGARCH (1, 1).

	GARCH	TGARCH	EGARCH	PARCH	CGARCH	GARCH	TGARCH	EGARCH	PARCH	CGARCH
Model	(1, 1)	(1, 1)	(1, 1)	(1, 1)	(1, 1)	(2, 2)	(2, 2)	(2, 2)	(2, 2)	(2, 2)
Akaike info criterion	7.276865	7.277845	7.326129	7.277096	7.564447	7.469013	7.501975	7.325462	7.507242	7.503717
Schwarz criterion	7.300820	7.305222	7.353506	7.307895	7.595246	7.499812	7.539618	7.363105	7.548308	7.537939
Hannan-Quinn criterion	7.285771	7.288022	7.336306	7.288545	7.575897	7.480462	7.515968	7.339456	7.522508	7.516439
Adjusted R-squared	0.933464	0.933490	0.931961	0.933896	0.955248	0.952755	0.934963	0.931726	0.956706	0.956799

Garch model achived under student's t distribution

Method: ML ARCH - Student's t distribution (BFGS / Marquardt steps)

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All the models are then compared using the smallest values of the Akaike information criterion (AIC) Hannan-Quinn criterion (HIC) and Schwarz information criterion (SIC) during estimation to select the best model which is better for exchange rate volatility if more than one model with given specifications is found to be significant.

Result discussion of Garch models under students t–distribution

From the above result comparison between the GARCH models under the Students *t* distribution, the selection criterion were based on Selecting the Minimum AIC, SIC and HIC, from the based result above Akaike Information Criterion, Schwarz Criterion and Hannan-Quinn Criterion that the best models have proved to be GARCH (1, 1). With the minimum estimated AIC of 7.276865, SIC estimated to be 7.300820, and HIC estimated to be 7.285771. Therefore, selecting the best model using the Students *t* distribution is the GARCH (1, 1).

Garch model achived under generalized error distribution (GED)

All the models are then compared using the smallest values of the Akaike information criterion (AIC) Hannan-Quinn criterion (HIC) and Schwarz information criterion (SIC) during estimation to select the best model which is better for exchange rate volatility if more than one model with given specifications is found to be significant.

Method: ML ARCH - Generalized error distribution (GED) (BFGS / Marquardt steps)

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	GARCH	TGARCH	EGARCH	PARCH	CGARCH	GARCH	TGARCH	EGARCH	PARCH	CGARCH
Model	(1, 1)	(1, 1)	(1, 1)	(1, 1)	(1, 1)	(2, 2)	(2, 2)	(2, 2)	(2, 2)	(2, 2)
Akaike info criterion	7.070348	7.067117	7.143296	6.894232	7.422652	7.153267	7.401367	6.939807	7.143557	7.160469
Schwarz criterion	7.094303	7.094494	7.170673	6.925031	7.453451	7.184066	7.439010	6.977451	7.184623	7.194690
Hannan-Quinn criterion	7.079253	7.077294	7.153473	6.905681	7.434102	7.164716	7.415361	6.953801	7.158823	7.173190
Adjusted R-squared	0.929565	0.929332	0.956608	0.921757	0.955741	0.951780	0.925898	0.926010	0.949771	0.956164

Result discussion of Garch models under generalized error distribution (GED)

From the above result comparison between the GARCH models under the Generalized error distribution (GED), the selection criterion were based on Selecting the Minimum AIC, SIC and HIC, from the based result above Akaike Information Criterion, Schwarz Criterion and Hannan-Quinn Criterion that the best models have proved to be PARCH (1, 1). With the minimum estimated AIC of 6.894232, SIC estimated to be 6.925031, and HIC estimated to be 6.905681. Therefore, selecting the best model using the Generalized error distribution is the PARCH (1, 1).

Conclusion

Based on the analyses and interpretations of the appropriate secondary data on exchange rate volatility in Nigeria obtained from central bank of Nigeria covering the period of 2nd January 2012 to 29th December 2017, the study revealed that the data used for the research work are all stationary, in addition the heteroscedasticity test indicates that there is no presence of ARCH effect left in the residual which proves a good sign for the GARCH model.

From the GARCH model achieved under three different error distributions, which are normal Gaussian distribution, Students *t* distribution and Generalized error distribution (GED) all the models are then compared using the minimum value of the Akaike information criterion (AIC), Hannan-Quinn criterion (HIC), and Schwarz information criterion

(SIC) to select the best model which is better for research work on exchange rate volatility in Nigeria. GARCH models under the normal Gaussian distribution, the best models have proved to be CGARCH (1, 1). GARCH models under the Students *t* distribution, the best models have proved to be GARCH (1, 1). Finally, GARCH models under the Generalized error distribution (GED), the best models have proved to be PARCH (1, 1). Based on computations and analysis carried out on daily exchange rate data, one can conclude that the naira exchange rate is highly volatile and responds significantly to information shock, results suggest existence of conditional heteroscedasticity or volatility clustering.

Conflict of Interest

The authors declare that there is no conflict of interest related to this study.

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